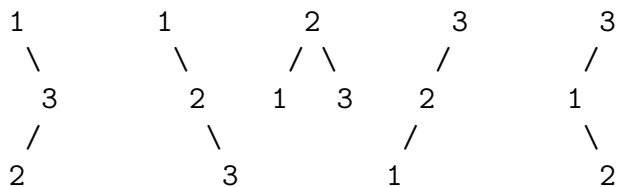


# 1 Climbing some trees...

**Problem 1:** Here's a really interesting problem, that connects trees to recursion, and the Catalan numbers in particular: Given  $n \in \mathbb{N}$ , find  $s(n)$  – the number of structurally unique binary search trees that store values 1 through  $n$ . Find the recursion relation for  $s(n)$ , and its first 8 values. (For convenience let's define  $s(0) = 1$ .) This figure shows that  $s(3) = 5$ :



Using the same quaint graphing technique, but emphasizing the symmetry, we see that there are five possible BSTs for  $n = 3$ :



Or maybe this representation is more suggestive (with the  $n = 2$  and  $n = 1$  cases thrown in for good measure):



**Problem 2:** To test blood from blood donors for coronavirus, small samples of blood from  $n = 2^m$  ( $m \in \mathbb{N}$ ) donors are pooled, and then **the pooled sample is tested**. If negative, great – all clear, after just one test; if positive, however, apply the strategy recursively on pooled samples of two halves, one after the other.

- Best case scenario:** Suppose we know that **only one person** is infected (**and the lab knows it, too**): how many Covid-19 tests will the lab do in order to identify the individual? (You might consider some simple cases.) How does that compare to just testing everyone?
- Worst case scenario:** Suppose we know that **all** are infected (**but the lab doesn't know it**). How many tests will the lab do in order to determine that? How does that compare to just testing everyone?
- Reality:** is generally somewhere between best and worst cases. At what prevalence (infection rate) will sequential testing of all donors require about the same number of tests as this divide-and-conquer strategy?