SECTION 1.1 REVIEW

TECHNIQUES

- Construct truth tables for compound wffs.
- Recognize tautologies and contradictions.

MAIN IDEAS

- Wffs are symbolic representations of statements.
- Truth values for compound wffs depend on the truth values of their components and the types of connectives used.
- Tautologies are "intrinsically true" wffs—true for all truth values.

EXERCISES 1.1

1. Which of the following sentences are statements?
   a. The moon is made of green cheese.
   b. He is certainly a tall man.
   c. Two is a prime number.
   d. The game will be over by 4:00.
   e. Next year interest rates will rise.
   f. Next year interest rates will fall.
   g. $x^2 - 4 = 0$
2. What is the truth value of each of the following statements?
   a. 8 is even or 6 is odd.
   b. 8 is even and 6 is odd.
   c. 8 is odd or 6 is odd.
   d. 8 is odd and 6 is odd.
   e. If 8 is odd, then 6 is odd.
   f. If 8 is even, then 6 is odd.
   g. If 8 is odd, then 6 is even.
   h. If 8 is odd and 6 is even, then $8 < 6$.
3. Given the truth values A true, B false, and C true, what is the truth value of each of the following wffs?
   a. $A \wedge (B \vee C)$
   b. $(A \wedge B) \vee C$
   c. $(A \wedge B)' \vee C$
   d. $A' \vee (B' \wedge C)'$
4. Given the truth values A false, B true, and C true, what is the truth value of each of the following wffs?
   a. $A \rightarrow (B \vee C)$
   b. $(A \vee B) \rightarrow C$
   c. $C \rightarrow (A' \wedge B')$
   d. $A \vee (B' \rightarrow C)$
5. Rewrite each of the following statements in the form "If A, then B."
   a. Healthy plant growth follows from sufficient water.
   b. Increased availability of information is a necessary condition for further technological advances.
   c. Errors were introduced only if there was a modification of the program.
   d. Fuel savings implies good insulation or storm windows throughout.

6. Rewrite each of the following statements in the form "If A, then B."
   a. Candidate Lu winning the election will be a sufficient condition for property taxes to increase.
   b. The user clicks Pause only if the game level changes.
   c. The components are scarce, therefore the price increases.
   d. Healthy hair is a necessary condition for good shampoo.
7. Common English has many ways to describe logical connectives. Write a wff for each of the following expressions.
   a. Either A or B
   b. Neither A nor B
8. Common English has many ways to describe logical connectives. Write a wff for each of the following expressions.
   a. B whenever A
   b. A is derived from B
   c. A indicates B
   d. A exactly when B
9. Several forms of negation are given for each of the following statements. Which are correct?
   a. The answer is either 2 or 3.
      1. Neither 2 nor 3 is the answer.
      2. The answer is not 2 or not 3.
      3. The answer is not 2 and it is not 3.
   b. Cucumbers are green and seedy.
      1. Cucumbers are not green and not seedy.
      2. Cucumbers are not green or not seedy.
      3. Cucumbers are green and not seedy.
   c. $2 < 7$ and 3 is odd.
      1. $2 > 7$ and 3 is even.
      2. $2 \geq 7$ and 3 is even.
      3. $2 \geq 7$ or 3 is odd.
      4. $2 \geq 7$ or 3 is even.
10. Several forms of negation are given for each of the following statements. Which are correct?
    a. The carton is sealed or the milk is sour.
       1. The milk is not sour or the carton is not sealed.
       2. The carton is not sealed and also the milk is not sour.
       3. If the carton is not sealed, then the milk will be sour.
    b. Flowers will bloom only if it rains.
       1. The flowers will bloom but it will not rain.
       2. The flowers will not bloom and it will not rain.
       3. The flowers will not bloom or else it will not rain.
    c. If you build it, they will come.
       1. If you build it, then they won't come.
       2. You don't build it, but they do come.
       3. You build it, but they don't come.
11. Write the negation of each statement.
    a. If the food is good, then the service is excellent.
    b. Either the food is good or the service is excellent.

c. Either the food is good and the service is excellent, or else the price is high.

d. Neither the food is good nor the service excellent.

e. If the price is high, then the food is good and the service is excellent.

12. Write the negation of each statement.

a. The processor is fast but the printer is slow.

b. The processor is fast or else the printer is slow.

c. If the processor is fast, then the printer is slow.

d. Either the processor is fast and the printer is slow, or else the file is damaged.

e. If the file is not damaged and the processor is fast, then the printer is slow.

f. The printer is slow only if the file is damaged.

13. Using the letters indicated for the component statements, translate the following compound statements into symbolic notation.

a. A: prices go up; B: housing will be plentiful; C: housing will be expensive
If prices go up, then housing will be plentiful and expensive; but if housing is not expensive, then it will still be plentiful.

b. A: going to bed; B: going swimming; C: changing clothes
Either going to bed or going swimming is a sufficient condition for changing clothes; however, changing clothes does not mean going swimming.

c. A: it will rain; B: it will snow
Either it will rain or it will snow but not both.

d. A: Janet wins; B: Janet loses; C: Janet will be tired
If Janet wins or if she loses, she will be tired.

e. A: Janet wins; B: Janet loses; C: Janet will be tired
Either Janet will win or, if she loses, she will be tired.

14. Using the letters indicated for the component statements, translate the following compound statements into symbolic notation.

a. A: the tractor wins; B: the truck wins; C: the race will be exciting.
Whether the tractor wins or the truck wins, the race will be exciting

b. A: snow; B: rain; C: yesterday was cloudy
Yesterday was cloudy but there was neither snow nor rain.

c. A: Koalas will be saved; B: climate change is addressed; C: rising water levels
Koalas will be saved only if climate change is addressed; furthermore, failure to address climate change will cause rising water levels.

d. A: the city's economy will improve; B: a strong school system
The city's economy will improve conditional upon a strong school system.

e. A: the city's economy will improve; B: a strong school system
A strong school system is a necessary condition for the city's economy to improve.

15. Let A, B, and C be the following statements:

A Roses are red.
B Violets are blue.
C Sugar is sweet.

Translate the following compound statements into symbolic notation.

a. Roses are red and violets are blue.

b. Roses are red, and either violets are blue or sugar is sweet.

c. Whenever violets are blue, roses are red and sugar is sweet.

d. Roses are red only if violets aren't blue or sugar is sour.

e. Roses are red and, if sugar is sour, then either violets aren't blue or sugar is sweet.

16. Let A, B, and C, and D be the following statements:

A The villain is French.

B The hero is American.

C The heroine is British.

D The movie is good.

Translate the following compound statements into symbolic notation.

a. The hero is American and the movie is good.

b. Athough the villain is French, the movie is good.

c. If the movie is good, then either the hero is American or the heroine is British.

d. The hero is not American, but the villain is French.

e. A British heroine is a necessary condition for the movie to be good.

17. Use A, B, and C as defined in Exercise 15 to translate the following statements into English.

a. $B \vee C'$

b. $B' \vee (A \rightarrow C)$

c. $(C \wedge A') \leftrightarrow B$

d. $C \wedge (A' \leftrightarrow B)$

e. $(B \wedge C')' \rightarrow A$

f. $A \vee (B \wedge C')$

g. $(A \vee B) \wedge C'$

18. Use A, B, and C as defined in Exercise 16 to translate the following statements into English.

a. $B \rightarrow A'$

b. $B \wedge C \wedge D'$

c. $B \rightarrow (C \vee A)$

d. $(A \vee C) \rightarrow B'$

e. $A \leftrightarrow (B \vee C)$

f. $D' \rightarrow (A \vee C)'$

g. $(C \rightarrow D) \wedge (A \rightarrow B')$

19. Using letters H, K, A for the component statements, translate the following compound statements into symbolic notation.

a. If the horse is fresh, then the knight will win.

b. The knight will win only if the horse is fresh and the armor is strong.

c. A fresh horse is a necessary condition for the knight to win.

d. The knight will win if and only if the armor is strong.

e. A sufficient condition for the knight to win is that the armor is strong or the horse is fresh.

20. Using letters A, T, E for the component statements, translate the following compound statements into symbolic notation.

a. If Anita wins the election, then tax rates will be reduced.

b. Tax rates will be reduced only if Anita wins the election and the economy remains strong.

c. Tax rates will be reduced if the economy remains strong.

d. A strong economy will follow from Anita winning the election.

e. The economy will remain strong if and only if Anita wins the election or tax rates are reduced.

21. Using letters F, B, S for the component statements, translate the following compound statements into symbolic notation.

a. Plentiful fish are a sufficient condition for bears to be happy.

b. Bears are happy only if there are plentiful fish.

c. Unhappy bears means that the fish are not plentiful and also that there is heavy snow.

d. Unhappy bears are a necessary condition for heavy snow.

e. The snow is heavy if and only if the fish are not plentiful.

22. Using letters P, C, B, L for the component statements, translate the following compound statements into symbolic notation.

a. If the project is finished soon, then the client will be happy and the bills will be paid.

b. If the bills are not paid, then the lights will go out.

c. The project will be finished soon only if the lights do not go out.

d. If the bills are not paid and the lights go out, then the client will not be happy.

e. The bills will be paid if and only if the project is finished soon, or else the lights go out.

f. The bills will be paid if and only if either the project is finished soon or the lights go out.

23. Construct truth tables for the following wffs. Note any tautologies or contradictions.

a. $(A \to B) \leftrightarrow A' \vee B$

b. $(A \wedge B) \vee C \to A \wedge (B \vee C)$

c. $A \wedge (A' \vee B')'$

d. $A \wedge B \to A'$

e. $(A \to B) \to [(A \vee C) \to (B \vee C)]$

24. Construct truth tables for the following wffs. Note any tautologies or contradictions.

a. $A \to (B \to A)$

b. $A \wedge B \leftrightarrow B' \vee A'$

c. $(A \vee B') \wedge (A \wedge B)'$

d. $[(A \vee B) \wedge C'] \to A' \vee C$

e. $A' \to (B \vee C')$

25. Verify the equivalences in the list on page 9 by constructing truth tables. (We have already verified 1a, 4b, and 5a.)

26. Verify by constructing truth tables that the following wffs are tautologies. Note that the tautologies in parts b, e, f, and g produce equivalences such as $(A')' \Leftrightarrow A$.

a. $A \vee A'$

b. $(A')' \leftrightarrow A$

c. $A \wedge B \to B$

d. $A \to A \vee B$

e. $(A \vee B)' \leftrightarrow A' \wedge B'$ (De Morgan's law)

f. $(A \wedge B)' \leftrightarrow A' \vee B'$ (De Morgan's law)

g. $A \vee A \leftrightarrow A$

27. Prove the following tautologies by starting with the left side and finding a series of equivalent wffs that will convert the left side into the right side. You may use any of the equivalencies in the list on page 9 or the equivalencies from Exercise 26.

a. $(A \wedge B') \wedge C \leftrightarrow (A \wedge C) \wedge B'$

b. $(A \vee B) \wedge (A \vee B') \leftrightarrow A$

c. $A \vee (B \wedge A') \leftrightarrow A \vee B$

28. Prove the following tautologies by starting with the left side and finding a series of equivalent wffs that will convert the left side into the right side. You may use any of the equivalencies in the list on page 9 or the equivalencies from Exercise 26.

a. $(A \wedge B')' \vee B \leftrightarrow A' \vee B$

b. $A \wedge (A \wedge B')' \leftrightarrow A \wedge B$

c. $(A \wedge B)' \wedge (A \vee B') \leftrightarrow B'$

29. We mentioned that $(A \wedge B) \vee C$ cannot be proved equivalent to $A \wedge (B \vee C)$ using either of the associative tautological equivalences, but perhaps it can be proved some other way. Are these two wffs equivalent? Prove or disprove.

30. Let P be the wff $A \rightarrow B$. Prove or disprove whether P is equivalent to any of the following related wffs.
    a. the *converse* of P, $B \rightarrow A$
    b. the *inverse* of P, $A' \rightarrow B'$
    c. the *contrapositive* of P, $B' \rightarrow A'$
31. Write a logical expression for a Web search engine to find sites pertaining to dogs that are not retrievers.
32. Write a logical expression for a Web search engine to find sites pertaining to oil paintings by Van Gogh or Rembrandt but not Vermeer.
33. Write a logical expression for a Web search engine to find sites pertaining to novels or plays about AIDS.
34. Write a logical expression for a Web search engine to find sites pertaining to coastal wetlands in Louisiana but not in Alabama.
35. Consider the following pseudocode.

```
repeat
    i = 1
    read a value for x
    if ((x < 5.0) and (2x < 10.7)) or (√5x > 5.1) then
        write the value of x
    end if
    increase i by 1
until i > 5
```

The input values for x are 1.0, 5.1, 2.4, 7.2, and 5.3. What are the output values?

36. Suppose that A, B, and C represent conditions that will be true or false when a certain computer program is executed. Suppose further that you want the program to carry out a certain task only when A or B is true (but not both) and C is false. Using A, B, and C and the connectives AND, OR, and NOT, write a statement that will be true only under these conditions.
37. Rewrite the following statement form with a simplified conditional expression, where the function *odd*(n) returns true if n is odd.

```
if not((Value1 < Value2) or odd(Number))
or (not(Value1 < Value2) and odd(Number)) then
    statement1
else
    statement2
end if
```

38. You want your program to execute statement 1 when A is false, B is false, and C is true, and to execute statement 2 otherwise. You wrote

```
if not(A and B) and C then
    statement 1
else
    statement 2
end if
```

Does this do what you want?

39. Verify that $A \rightarrow B$ is equivalent to $A' \vee B$.
40. a. Using Exercise 39 and other equivalences, prove that the negation of $A \rightarrow B$ is equivalent to $A \wedge B'$
    b. Write the negation of the statement "If Sam passed his bar exam, then he will get the job."

41. Use algorithm *TautologyTest* to prove that the following expressions are tautologies.
   a. $[B' \wedge (A \rightarrow B)] \rightarrow A'$
   b. $[(A \rightarrow B) \wedge A] \rightarrow B$
   c. $(A \vee B) \wedge A' \rightarrow B$
42. Use algorithm *TautologyTest* to prove that the following expressions are tautologies.
   a. $(A \wedge B) \wedge B' \rightarrow A$
   b. $(A \wedge B') \rightarrow (A \rightarrow B)'$
   c. $(A \wedge B)' \vee B' \rightarrow A' \vee B'$
43. A memory chip from a digital camera has 2^5 bistable (ON-OFF) memory elements. What is the total number of ON-OFF configurations?
44. In each case, construct compound wffs P and Q so that the given statement is a tautology.
   a. $P \wedge Q$
   b. $P \rightarrow P'$
   c. $P \wedge (Q \rightarrow P')$
45. From the truth table for $A \vee B$, the value of $A \vee B$ is true if A is true, if B is true, or if both are true. This use of the word "or," where the result is true if both components are true, is called the *inclusive or*. It is the inclusive or that is understood in the sentence, "We may have rain or drizzle tomorrow," which might also be expressed as, "We may have rain or drizzle or both tomorrow." Another use of the word "or" in the English language is the *exclusive or*, sometimes written **XOR**, in which the result is false when both components are true. The exclusive or is understood in the sentence, "At the intersection, you should turn north or south," (but obviously not both). Exclusive or is symbolized by $A \oplus B$. Write the truth table for the exclusive or.
46. Prove that $A \oplus B \leftrightarrow (A \leftrightarrow B)'$ is a tautology. Explain why this makes sense.

Exercises 47–50 show that defining four basic logical connectives (conjunction, disjunction, implication, and negation) is a convenience rather than a necessity because certain pairs of connectives are enough to express any wff. Exercises 51–52 show that a single connective, properly defined, is sufficient.

47. Every compound statement is equivalent to a statement using only the connectives of conjunction and negation. To see this, we need to find equivalent wffs for $A \vee B$ and for $A \rightarrow B$ that use only $\wedge$ and $'$. These new statements can replace, respectively, any occurrences of $A \vee B$ and $A \rightarrow B$. (The connective $\leftrightarrow$ was defined in terms of other connectives, so we already know that it can be replaced by a statement using these other connectives.)
   a. Show that $A \vee B$ is equivalent to $(A' \wedge B')'$
   b. Show that $A \rightarrow B$ is equivalent to $(A \wedge B')'$
48. Show that every compound wff is equivalent to a wff using only the connectives of $\vee$ and $'$. (*Hint*: See Exercise 47.)
49. Show that every compound wff is equivalent to a wff using only the connectives of $\rightarrow$ and $'$. (*Hint*: See Exercise 47.)
50. Prove that there are compound statements that are not equivalent to any statement using only the connectives $\rightarrow$ and $\vee$.
51. The binary connective $|$ is called the *Sheffer stroke*, named for the American logic professor Henry Sheffer, who proved in 1913 that this single connective is the only one needed. The truth table for $|$ is given here. Sheffer also coined the term "Boolean algebra," the topic of Chapter 8, where we will see that this truth table represents the NAND gate.

A	B	$A \mid B$
T	T	F
T	F	T
F	T	T
F	F	T

Show that every compound wff is equivalent to a wff using only the connective $\mid$. (*Hint*: Use Exercise 47 and find equivalent statements for $A \wedge B$ and A' in terms of $\mid$.)

52. The binary connective $\downarrow$ is called the *Peirce arrow*, named for American philosopher Charles Peirce (not for the antique automobile). The truth table for $\downarrow$ is given here. In Chapter 8 we will see that this truth table represents the NOR gate

A	B	$A \downarrow B$
T	T	F
T	F	F
F	T	F
F	F	T

Show that every compound statement is equivalent to a statement using only the connective $\downarrow$. (*Hint*: See Exercise 51.)

53. Propositional wffs and truth tables belong to a system of *two-valued logic* because everything has one of two values, False or True. *Three-valued logic* allows a third value of Null or "unknown" (Section 5.3 discusses the implications of three-valued logic on databases). The truth tables for this three-valued system follow.

A	B	$A \wedge B$
T	T	T
T	F	F
T	N	N
F	T	F
F	F	F
F	N	F
N	T	N
N	F	F
N	N	N

A	B	$A \vee B$
T	T	T
T	F	T
T	N	T
F	T	T
F	F	F
F	N	N
N	T	T
N	F	N
N	N	N

A	A′
T	F
F	T
N	N

a. Viewing N as "unknown", explain why it is reasonable to define $T \wedge N = N$, $F \vee N = N$, and $N' = N$.

Suppose the statement, "Flight 237 is on time," is true, the statement, "Runway conditions are icy," is false, and the truth value of the statement, "Flight 51 is on time," is unknown. Find the truth values of the following statements.

b. Runway conditions are not icy and flight 51 is on time.

c. Flight 51 is on time and flight 237 is not.

d. Flight 51 is not on time or runway conditions are not icy.

54. Propositional wffs and truth tables belong to a system of *two-valued logic* because everything has one of two values, F or T, which we can think of as 0 or 1. In *fuzzy logic*, or *many-valued logic*, statement letters are assigned values in a range between 0 and 1 to reflect some "probability" to which they are false or true. A statement letter with a truth value of 0.9 is "mostly true" or "has a high probability of being true" while a statement letter with a truth value of 0.05 "has a very high probability of being false." Fuzzy logic

is used to manage decisions in many imprecise situations such as robotics, manufacturing, or instrument control. Truth values for compound statements are determined as follows.

A' has the truth value $1 - A$.
$A \wedge B$ has the truth value that is the minimum of the values of A and of B.
$A \vee B$ has the truth value that is the maximum of the values of A and B.

a. Explain why these are reasonable assignments for the truth values of A', $A \wedge B$, and $A \vee B$.

Suppose the statement, "Flight 237 is on time," is estimated to have a truth value of 0.84 and the statement, "Runway conditions are icy," is estimated to have a truth value of 0.12. Find the truth values of the following statements.

b. Runway conditions are not icy.

c. Runway conditions are icy and flight 237 is on time.

d. Runway conditions are icy or flight 237 is not on time.

55. In a three-valued logic system as described in Exercise 53, how many rows are needed for a truth table with n statement letters?

56. In 2003, then U.S. Secretary of Defense Donald Rumsfeld won Britain's Plain English Campaign 2003 Golden Bull Award for this statement: "Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns, there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns—the ones we don't know we don't know."

What possibility did Secretary Rumsfeld omit?

57. Four machines, A, B, C, and D, are connected on a computer network. It is feared that a computer virus may have infected the network. Your security team makes the following statements:

a. If D is infected, then so is C.

b. If C is infected, then so is A.

c. If D is clean, then B is clean but C is infected.

d. If A is infected, then either B is infected or C is clean.

Assuming that these statements are all true, what can you conclude? Explain your reasoning.

58. The Dillies have five teenaged children, two boys named Ollie and Rollie, and three girls named Mellie, Nellie, and Pollie. Each is a different number of years old, from 13 to 17. There are three bedrooms for the children in the Dillie house, so two share the yellow room, two share the white room, and one alone has the smaller green room. Can you match each one's name and age, and tell who sleeps where?

a. No one shares a room with a sibling of the opposite sex.

b. Pollie is exactly one year older than Mellie.

c. The two teenagers who share the yellow room are two years apart in age.

d. The two who share the white room are three years apart in age.

e. Rollie is somewhat older than Ollie but somewhat younger than the sibling who has the green room.[2]

Determine who sleeps in each room and what their ages are. Explain your reasoning.

59. An advertisement for a restaurant at an exclusive club in Honolulu says, "Members and nonmembers only." Give two possible interpretations of this statement.

60. The following newspaper headline was printed during a murder trial:

"I am a liar" says murder defendant!

Can the jury reach any conclusion from this statement?

[2]Scott Marley, Dell Logic Puzzles, April, 1998

In Exercises 61–64, you are traveling in a certain country where every inhabitant is either a truthteller who always tells the truth or a liar who always lies.[3]

61. You meet two of the inhabitants of this country, Percival and Llewellyn. Percival says, "At least one of us is a liar." Is Percival a liar or a truth teller? What about Llewellyn? Explain your answer.
62. Traveling on, you meet Merlin and Meredith. Merlin says, "If I am a truth teller, then Meredith is a truth teller." Is Merlin a liar or a truth teller? What about Meredith? Explain your answer.
63. Next, you meet Rothwold and Grymlin. Rothwold says, "Either I am a liar or Grymlin is a truth teller." Is Rothwold a liar or a truth teller? What about Grymlin? Explain your answer.
64. Finally, you meet Gwendolyn and Merrilaine. Gwendolin says, "I am a liar but Merrilaine is not." Is Gwendolyn a liar or a truth teller? What about Merrilaine?

SECTION 1.2 PROPOSITIONAL LOGIC

The argument of the defense attorney at the beginning of this chapter made a number of (supposedly true) statements and then asked the jury to draw a specific conclusion based on those statements. In Section 1.1, we used the notation of formal logic to represent statements in symbolic form as wffs; because statements are sometimes called *propositions*, these wffs are also called **propositional wffs.** Now we want to use tools from formal logic to see how to reach logical conclusions based on given statements. The formal system that uses propositional wffs is called **propositional logic**, **statement logic**, or **propositional calculus**. (The word *calculus* is used here in the more general sense of "calculation" or "reasoning," not "differentiating" or "integrating.")

Valid Arguments

An argument can be represented in symbolic form as

$$P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n \rightarrow Q$$

where $P_1, P_2, \ldots, P_n$ are the given statements, called the **hypotheses,** of the argument, and Q is the **conclusion** of the argument. As usual, the P's and the Q represent wffs, not merely statement letters. When should this be considered a *valid argument*? This question can be stated in several equivalent ways:

- When can Q be *logically deduced from* $P_1, \ldots, P_n$?
- When is Q a *logical conclusion from* $P_1, \ldots, P_n$?
- When does $P_1, \ldots, P_n$ *logically imply* Q?
- When does Q *follow logically from* $P_1, \ldots, P_n$?

and so forth.

[3]For more puzzles about "knights" and "knaves," see *What Is the Name of This Book?* by the logician—and magician—Raymond Smullyan (Prentice-Hall, 1978).