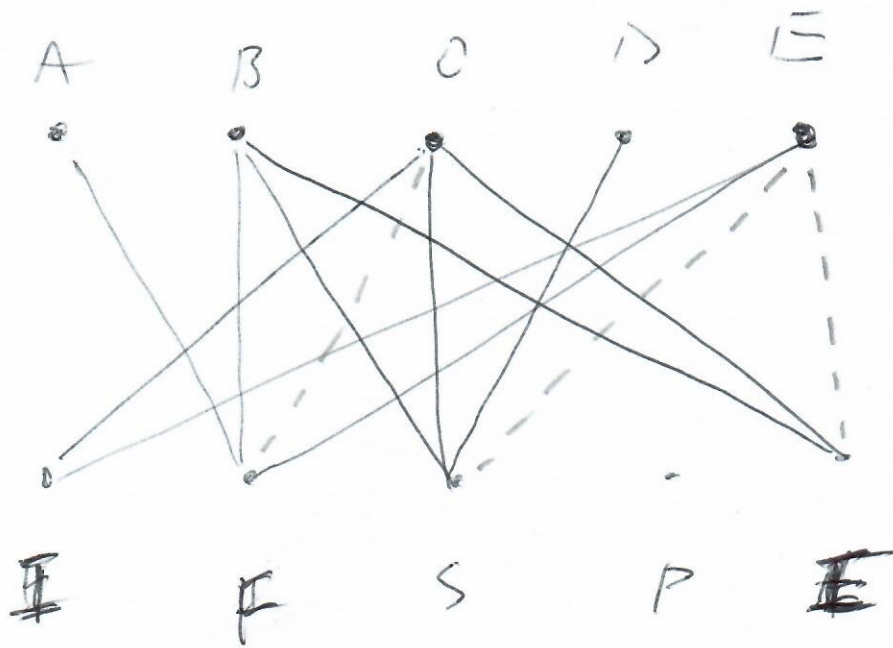


# Bipartite Graph (25 possible arcs)



Of 15 arcs  
totally  
10 accounted  
for with

#3, 4, 5

Now these can  
eliminate  
some, as  
well.

Let's use #3

There will be  $1+2+3+4+5$  arcs  
 $= 15$

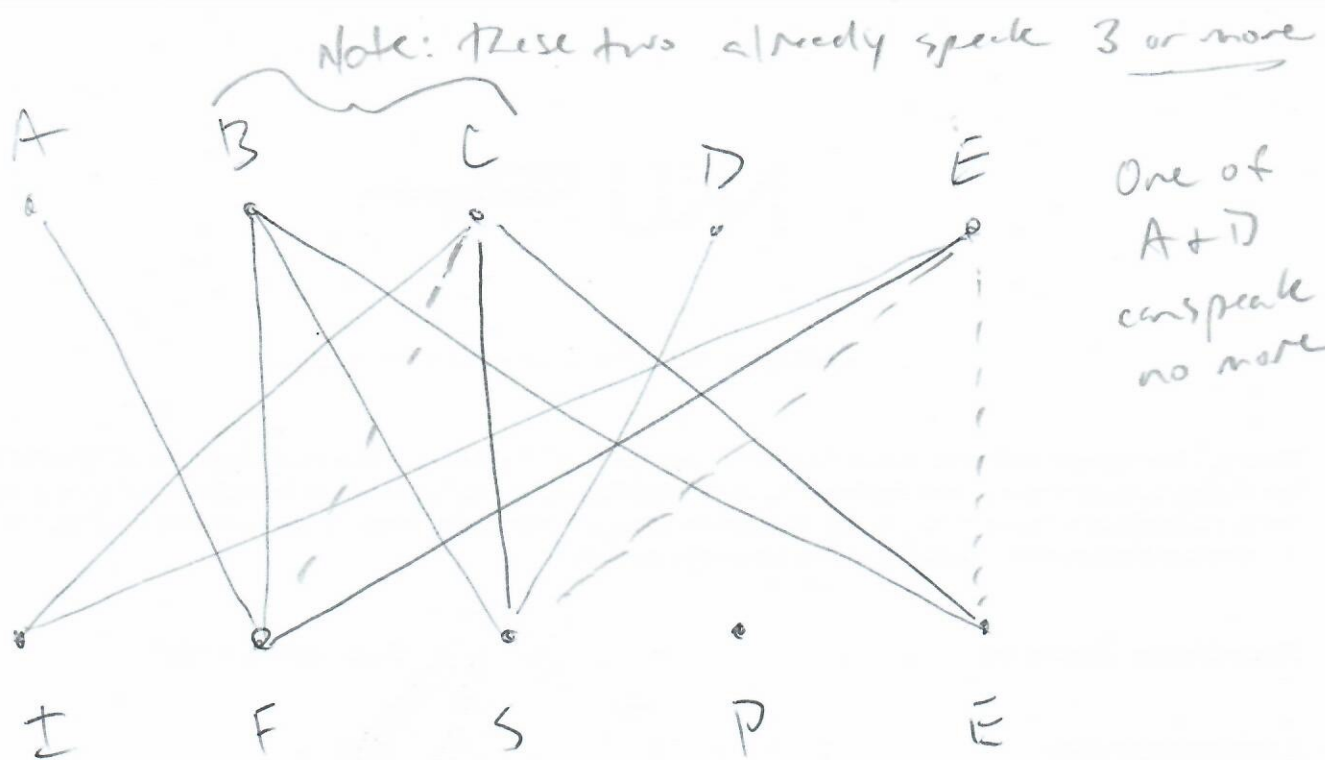
#6

The "most common language": if not at least 4, then all languages are spoken by 3, because

#2  
the sum spoken =  $\sum$  languages = 15 & there are 5 languages  $\rightarrow 15 = 5 \cdot 3$ . [If any one language were to drop below 3, then another would have to rise above 3!]

So: are all languages spoken by 3 people?

Could a language be spoken by all 5? If not, then "most common"  $\rightarrow$  ~~4~~ <sup>spoken by 5</sup> ~~3~~ 4



ABE cannot all speak Portuguese  
 CE cannot both " "

Three must speak Portuguese

Since one of A+D can speak no more, the choice for Portuguese is among

BCDE  $\begin{cases} BCD \times \rightarrow \text{~~2 4 4 2 2~~} \\ BCE \times \end{cases}$   
 or  
 ABCE  $\begin{cases} ABE \times \\ ABC \end{cases}$  2 4 4 (1) 2

There will be only two crew left. P, S, F all have three speakers; I + E only two.  
 A or  $\times \rightarrow 1$  (C has them)  
 B or  $\times \rightarrow 1$  (E has it)

