1. (2 pts) Explain how mathematical induction is similar to an infinite application of modus ponens.

→ During induction, we first find if the base case is true and then imply if P(m) is true then P(m+1) is true. By assuming P(m) is true we try to deduce P(m+1).

This is similar to first finding P(1) is true and then imply P(1) → P(2) and by modus ponens P(2) will be true and then continue to say P(2) → P(3), P(3) is true by modus ponens infinitely for all n. Just like a system of dominos.

Well done

2. (2 pts) Explain the difference between the first and second principles of mathematical induction.

→ With first principle of mathematical induction, we say if P(m), then P(m+1). By assuming P(m) to be true we try to find if P(m+1) is true.

→ With second principle of induction, we assume everything from base case to 'm' is true and deduce P(m+1) with our assumption.

Good

3. (6 pts) Prove that the sum of the first n odd natural numbers is n^2, for all natural numbers n.

odd numbers : $2n-1$, for $n \geq 1$

$1 + 3 + 5 + 7 + \cdots + 2n - 1 = n^2$

$P(n)$: $\sum_{i=1}^{n}(2i-1) = n^2$

Base Case : $P(1)$; $\sum_{1}^{1} 2i-1 = 2-1 = 1 = 1 = (1)^2$

Inductive Step: $P(m) \rightarrow P(m+1)$ is true.

Assume $P(m) = \sum_{i=1}^{m}(2i-1) = m^2$

Consider $P(m+1)$: $\sum_{i=1}^{m+1}(2i-1) = (m+1)^2$

Consider left-side :

$$\sum_{i=1}^{m+1}(2i-1) = \underbrace{\sum_{i=1}^{m}(2i-1)}_{P(n)} + 2(m+1) - 1$$

$$= m^2 + 2m + 2 - 1 = m^2 + 2m + 1$$

$$\sum_{i=1}^{m+1}(2i-1) = (m+1)^2$$

$\therefore (\forall n)\ P(n)$ is true by first principle of induction.

Very nice.

3. (6 pts) Prove that the sum of the first n odd natural numbers is n^2, for all natural numbers n.

1 2 3 4 5 6 7

odds: 1, 3, 5, 7, 9, 11, 13

$S(1) = 1 \quad 1^2 = 1 \quad 1 = 1$

$S(2) = 1+3 = 4 \quad 2^2 = 4 \quad 4 = 4$

$S(3) \quad 1+3+5 = 9 \quad 3^2 = 9 \quad 9 = 9$

Assume $S(n)$: $\sum_{i=1}^{n} ((2i)-1) = n^2$

Show: $S(n+1)$: $\sum_{i=1}^{n+1} ((2i)-1) = (n+1)^2$

$$\sum_{i=1}^{n+1} [(2i)-1] = \sum_{i=1}^{n} [(2i)-1] + [2(n+1)-1]$$

$$= \left[\sum_{i=1}^{n} ((2i)-1)\right] + [2n+2-1]$$

$$= [n^2] + (2n+1)$$

$$= n^2 + 2n + 1$$

$$= (n+1)^2$$

Thus

$$\sum_{i=1}^{n+1} (2i)-1 = (n+1)^2$$

Thus the sum of the first n odd numbers is n^2 for all n.

$\square$

3. (6 pts) Prove that the sum of the first n odd natural numbers is n^2, for all natural numbers n.

$P(n): \sum_{i=1}^{n} 2i-1 = n^2$

Base case: $P(1) = \sum_{i=1}^{1} 2(1)-1 = 1 = (1)^2$ ✓

Induction step: Assume $P(m) \to P(m+1)$

$P(m) \stackrel{?}{=} \sum_{i=1}^{m} 2i-1 = m^2$

$P(m+1) \stackrel{?}{=} \sum_{i=1}^{m+1} 2i-1 = (m+1)^2$

consider the left:

$P(m+1) \stackrel{?}{=} \sum_{i=1}^{m+1} 2i-1 = m^2 + 2(m+1)-1$

$= m^2 + 2m + 2 - 1$

$= m^2 + 2m + 1$

$= m^2 + m + m + 1$

$= m(m+1) + 1(m+1)$

$= (m+1)(m+1)$

$= (m+1)^2$ ✓

might include intermediate step!

$\sum_{i=1}^{m+1}(2i-1) = \underbrace{\sum_{i=1}^{m}(2i-1)}_{m^2} + \underbrace{2(m+1)-1}_{2m+1}$

$\therefore (\forall n)\ P(n)$ is true

✓