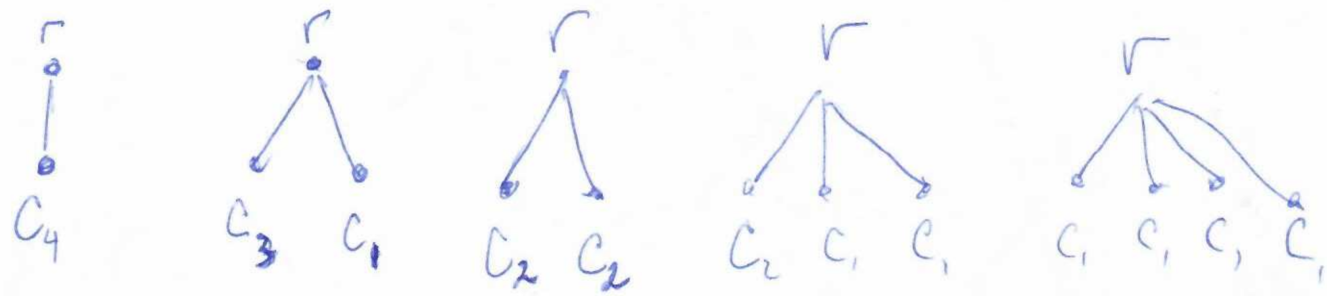
MAT385 Quiz 09, Spring 2023

Name:

1. (6 pts) Draw all the nonisomorphic trees with five nodes. Call the set C_5.



4 2 1 1

same non-iso? No: Just one way....

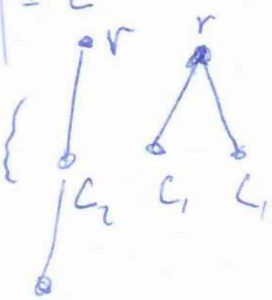
$|C_1| = 1$

$C_1 = \{ \bullet^r \}$

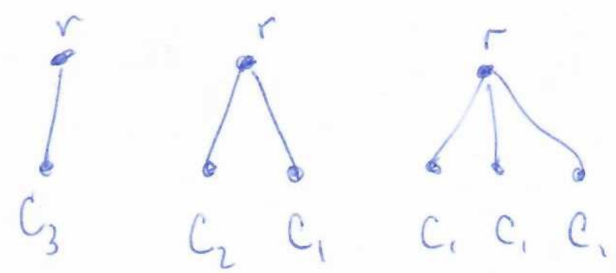
$|C_2| = 1$

$C_2 = \{$  $\}$

$|C_3| = 2$

$C_3 = \{$  $\}$

$|C_4| = 4$

$C_4 = \{$  $\}$

2 1 1

$|C_5| = \underline{\underline{9}}$ nonisomorphic trees w/ five nodes :

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Nice work

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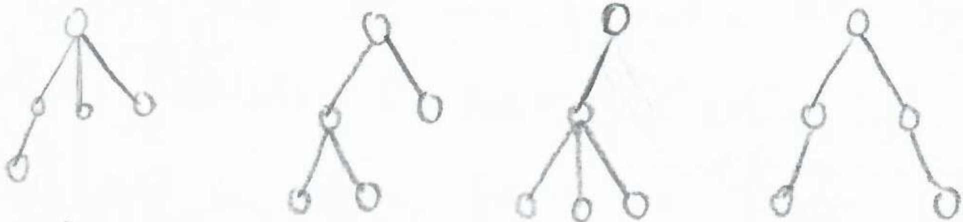
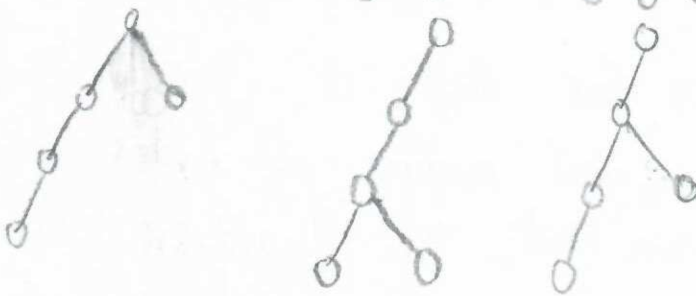
d (depth)

1

2



3



4

Excellent!

2. (4 pts) Prove that a binary tree has at most 2^d nodes at depth d. (Induction works.)

$P(d)$: a binary tree at depth d has at most 2^d nodes: $n(d) \le 2^d$.

Base case: every binary tree has just a root node at depth 0, the root. Hence $P(0)$ ✓: $1 \le 2^0 = 1$.

Inductive Step: Assume $P(k)$, & consider the $(k+1)$ depth of a binary tree, where it has $n(k+1)$ nodes. These nodes are the children of nodes at the k depth; &, since this is a binary tree, every node as at most 2 children. There are

$$n(k) \le 2^k$$

nodes at depth k by assumption, & the number of their children is ✓

$$n(k+1) \le n(k) \cdot 2 \le 2^{k+1} \therefore P(k+1).$$

By the 1^{st} principle of induction we have established $P(n)$ $\forall n \ge 0$.