MAT385 Quiz 10, Spring 2025

Name:

There are three questions (one on the back).

1. (5 pts) There are five properties (called axioms) that every Boolean algebra satisfies. There are two dual sets of these properties for the operations of $+, \cdot, '$ and for the special elements 0 and 1. Write down these properties (one of the two dual sets suffices):

   Hint: You might think about propositional logic, and set theory: their binary and unary operations follow the same five properties.

   a. Commutativity:

   b. Associativity:

   c. Distributivity:

   d. Complements:

   e. Identities:

2. (2 pts) The universal bound property says $x + 1 = 1$.

   a. Prove this, using the complements property from above, and idempotence.

   b. What is the dual of the universal bound property, that you get "for free"?

3. (3 pts) Prove idempotence: $x \cdot x = x$ (again, begin by using the complements property). I'll get you started:

   $x = x \cdot 1$