Problem 1. Propositional Logic

a. (6 pts) Using propositional logic, prove the following argument:

If school costs rise, the student population will fall. Either costs will rise, or the government will subsidize the schools. The school population does not decline. Therefore the government subsidized the schools.

Use the statement letters C (costs rise), P (population falls), G (government subsidizes).

b. (4 pts) Prove that

$$P \wedge P' -> Q$$

showing that from a contradiction anything follows. **Prove**: do not simply **assert**!

Problem 2. Using predicate logic,

a. (5 pts) write the following as a theorem, and then

b. (5 pts) prove it:

Every member of the board comes from industry or government. Everyone from government who has a law degree is in favor of the motion. John is not from industry, but he does have a law degree. Therefore, if John is a member of the board, he is in favor of the motion.

Use letters M(x), I(x), G(x), L(x), F(x), j – member, industry, government, law, favor, john.

Problem 3. Prove that the proposition

$$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots n \cdot 2^n = (n-1)2^{n+1} + 2$$

is true for every natural number n.

Problem 4.

a. (4 pts) For a set of 12 items, what is the minimum worst-case number of comparisons a search algorithm must do?

b. (4 pts) Given the set of months of the year, find an order in which to enter them so that the corresponding binary search tree has a minimum depth, and do so.

c. (2 pts) In what order would you insert them so as to obtain the worst binary search tree possible?

Problem 5. Given the infix expression $(a + b) * c - ((d - 3) + e/7)/(f + g)$.

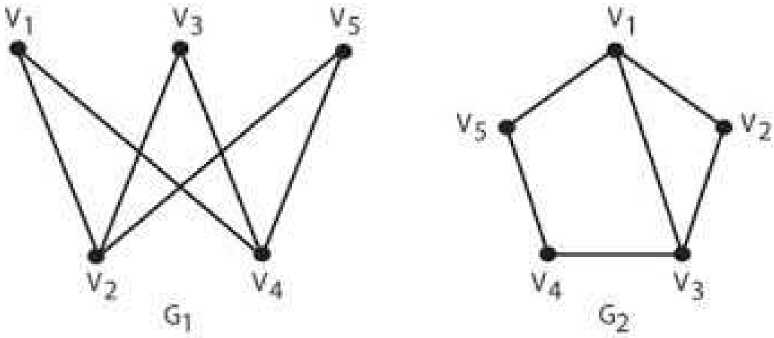
a. (6 pts) Draw the corresponding expression tree. What is its depth?

b. (4 pts) Give the equivalent postfix and prefix expressions, explaining how each works:

- postfix
- prefix

Problem 6. Isomorphisms and bijections (one-to-one and onto mappings).

a. (4 pts) The two graphs shown are not isomorphic: identify a characteristic of the graphs which demonstrates that they are not isomorphic.



b. (4 pts) If you add enough edges to each, you'll get two complete simple graphs, which are isomorphic. What is the **minimal number** of edges you must add to create isomorphic graphs? Justify!

c. (2 pts) If there is a bijection between the set of natural numbers and the set of prime numbers, what does this prove?

Problem 7. Suppose that performing an operation on a list of size n (assume $n = 2^m$ for m a natural number) requires three times the number of comparisons of performing the operation on a list of half that size, plus one comparison.

Suppose 1 comparison is required for a list of 1 element, i.e. $C(1) = 1$.

a. (2 pts) Write the first five terms of C, for lists of size 1, 2, 4, 8, and 16.

n	$C(n)$
1	
2	
4	
8	
16	

b. (4 pts) write the recurrence relation for $C(n)$ as a function of $C(n/2)$.

c. (4 pts) Use any method (formula or induction) to give a closed form solution for $C(n)$.

Problem 8:

a. (4 pts) Draw all 11 non-isomorphic simple graphs of four (unlabeled) vertices in an organized fashion. Label them (from A to K) by number of edges (smallest to largest).

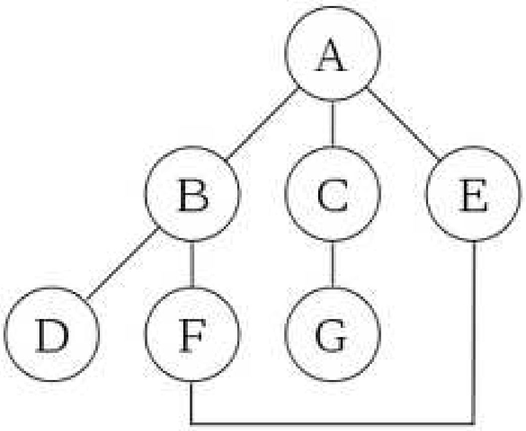
b. (3 pts) How many and which ones are connected and admit Euler paths? Define Euler paths.

c. (3 pts) How many and which ones admit Hamiltonian circuits? Define Hamiltonian circuits.

Problem 9:

a. (7 pts) Here is the Adjacency matrix for a graph G. Use Dijkstra's algorithm to compute the shortest distance from node a to node f:

	a	b	c	d	e	f
a	0	3	8	4	∞	10
b	3	0	∞	6	∞	∞
c	8	∞	0	∞	7	∞
d	4	6	∞	0	1	3
e	∞	∞	7	1	0	1
f	10	∞	∞	3	1	0



b. (3 pts) Find and draw a minimal spanning tree for G.

Problem 10: For the graph given in the following figure,

a. Illustrate a depth-first traversal of the graph starting from node f (note: the word **"Illustrate" means more than writing down the solution**).

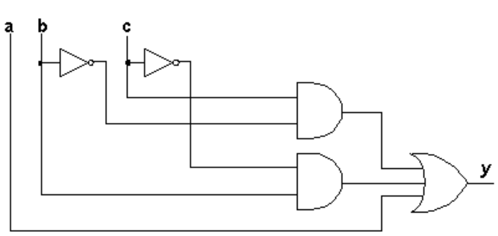
b. Illustrate a breadth-first traversal of the graph starting from node f.

Problem 11: Prove the following two properties of a Boolean Algebra:

a. $(x + y) \cdot (x + 1) = x + (x \cdot y) + y$

b. $(x \cdot y) + (x \cdot 0) = x \cdot (x + y) \cdot y$

Problem 12: For the logic network in the following figure



a. construct a Boolean expression (a sum of products), and

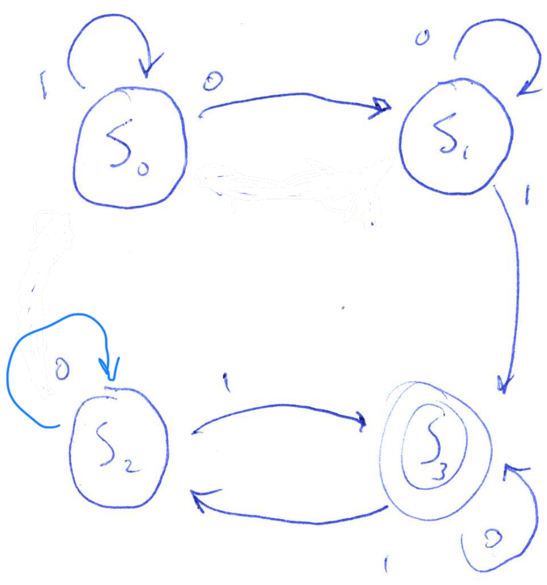
b. construct the truth table.

c. Then use a Karnough map to show that the expression is minimized.

Problem 13: Use Quine-McCluskey to find a minimal sum of products form for the Boolean expression

$$x_1x_2x_3x_4 + x_1x_2^{'}x_3x_4 + x_1x_2^{'}x_3x_4^{'} + x_1^{'}x_2x_3x_4 + x_1x_2x_3^{'}x_4 + x_1^{'}x_2x_3^{'}x_4$$

Problem 14: For the FSM given by the following State graph,



a. create the State Table.

b. write a regular expression that captures all recognized strings.

Problem 15: Create

a. a regular expression for all strings whose first 0 is followed by an even number of 11 pairs.

b. a finite state machine that recognizes such strings.