

MAT430 Exam 1 (Spring 2010)

Name:

Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

Problem 1. (10 pts)

- Express $z = \frac{2 - 3i}{i(2i - 3)}$ in the form $x + iy$ (and please – show all work!).

- Express $z = (\sqrt{3} - i)^4$ in the form $z = re^{i\theta}$ (show all work, and give all possible values of r and θ).

Problem 2. (10 pts) Sketch the set of points satisfying the condition $\text{Im}(\bar{z} - i) = 2$.

Problem 3. (10 pts) Find **all** the fourth roots of $z = -8 - 8\sqrt{3}i$, exhibit them geometrically, and point out which is the principal root.

Problem 4. (10 pts) Use Euler's and/or de Moivre's formula to derive identities for:

- $\cos(2\theta)$

- $\cos(\theta/2)$

- $\cos(\theta_1 + \theta_2)$

Problem 6. (10 pts)

- Suppose that you want to rotate objects in the plane by an angle of $\frac{-\pi}{2}$, and double the linear dimensions. What mapping by a complex function would you propose to accomplish this?

- Prove that $\lim_{z \rightarrow z_0} \frac{\bar{z}}{z}$ does not exist.