MAT430 Exam 3 (Spring 2010)

Name:

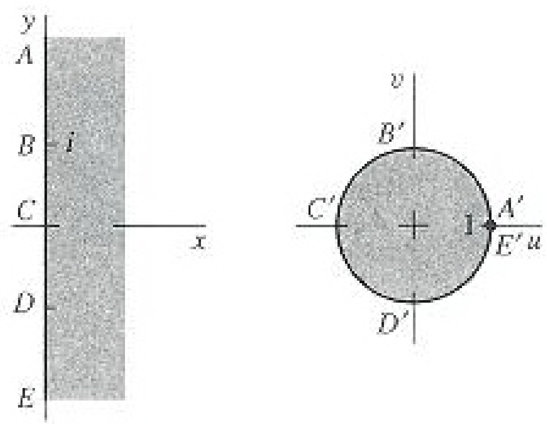
Directions: Show your work! Answers without justification will likely result in few points. Your written work also allows me the option of giving you partial credit in the event of an incorrect final answer (but good reasoning). Indicate clearly your answer to each problem (e.g., put a box around it). **Good luck!**

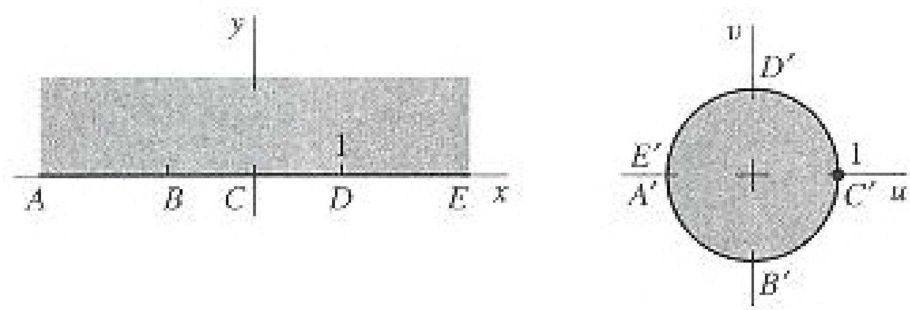
In-Class Portion: Do three of the first four problems (your choice), plus problem 5. Each should be on a separate page. You need only write the problem number at the top of the page.

Problem 1. The two transformations below belong to the mobius transformations

$$w = \frac{z-1}{z+1} \qquad \text{and} \qquad w = \frac{i-z}{i+z}$$

Which belongs with which? Give reasons.





Problem 2. Recall that a mobius transformation transforms straight lines and circles into straight lines and circles. Illustrate this fact for the mapping

$$w = \frac{i}{z}$$

(Can you simplify the problem at all?)

Problem 3. Without calculating the integral, bound

$$|\int_0^{\pi} e^{i\theta} d\theta|$$

Problem 4. Evaluate

$$\int_0^{\infty} e^{-zt} dt \qquad\qquad Re(z) > 0$$

Why is there a restriction on $Re(z)$?

Problem 5 (worth double): Refer to figure 18 below:

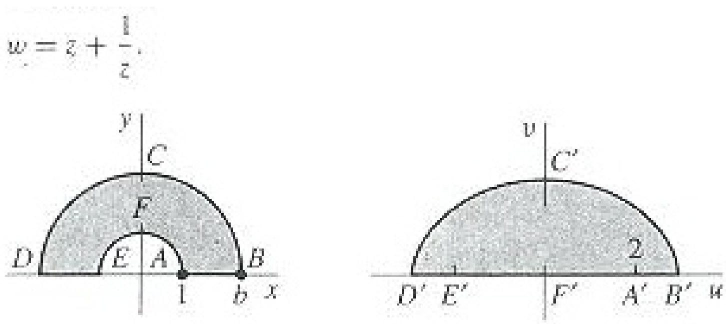


FIGURE 18
$w = z + \frac{1}{z}$; $B'C'D'$ on ellipse $\frac{u^2}{(b+1/b)^2} + \frac{v^2}{(b-1/b)^2} = 1$.

a. You have a steady state temperature distribution on the figure at right, with $b = 2$. Find B' and C'.

b. Assume that $T = 0$ on segment $E'A'$; $\frac{\partial T}{\partial n} = 0$ on $A'B'$ and $D'E'$; and $T = 1$ on the elliptical part of the region, $B'C'D'$. How would you go about using this diagram, and the process of conformal mapping, to find the isotherms and lines of flow on the region?

c. What can we say about the steady state temperature distribution T on the figure on the left?

"Take-Home" portion (due 4/23, at class time): You wish to determine the temperature distribution on the region between two circles, one inside the other:

a. C_1: center: $\frac{cosh(1)}{sinh(1)}$; radius: $\frac{1}{sinh(1)}$

b. C_2: center: $\frac{cosh(2)}{sinh(2)}$; radius: $\frac{1}{sinh(2)}$

The temperature on C_1 is 0, and the temperature on C_2 is 1; there is an insulating barrier joining the two left-most points of intersection of the two circles with the x-axis.

See if you can find the isotherms and lines of flow (Use the text's appendix!).