Packet 4: Hypothesis Test for the Population Proportion Textbook pages: 450 – 461; 516 – 523

After completing this material, you should be able to:

- conduct a test of hypothesis about p using the appropriate format.
- state when it is valid to use this procedure.
- define Type I and Type II errors in terms of the problem.
- discuss the consequences of these errors.

What is a hypothesis test? A statistical inference used to test a claim – conjecture – about a population parameter, based on data from a sample of the population.

Steps in a Hypothesis Test

Step 1: State the hypotheses to be tested.

The null hypothesis $H_0: p = \_$ is set up to be tested/attacked! We test it generally because we don't believe it: we have an alternative hypothesis H_a

Notation Alert!!

Null hypothesis: $H_0: p = \_$

Alternative hypothesis: H_a: (one of three)

$p > \_$
$p < \_$
$p \neq \_$

Significance level: $\alpha = \_$

Step 2: Set the significance level for the test.

The significance level α determines what we consider extreme. α determines how difficult it will be to reject the null H_0 in favor of the alternative H_a.

$\alpha = .05$
$(.01, .10)$

Step 3: Calculate the test statistic and p-value based upon the sample collected.

Test statistic is $z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

where $\hat{p}$ is sample proportion

Probability of something that extreme or more so

Draw the corresponding area(s)

Formula Alert!! This formula will be given on the formula sheet.

Step 4: Make a decision and interpret the results.

We are on the lookout for extreme values of the test statistic, which will have small probabilities associated with them. If the probability of a z-score as extreme (or more so) than we obtain is $< \alpha$, then we reject the null in favor of our alternative. Otherwise we fail to reject.

$p = .38$ $\quad np = 1066 \cdot .38 > 10$

$n(1-p) = 1066 \cdot .62 > 10$

Example 1: According to a 2014 report, 38% of all US teenagers (ages 13 – 17) report regularly using Snapchat. A social scientist believes this percentage has likely increased. To investigate the claim, a simple random sample of teenagers is taken from the United States Postal Service's database of addresses. Any selected household that contained a teenager who was willing to participate in the survey was included in the sample. The sample resulted in 1,066 teenagers of which 438 reported regularly using Snapchat. Use the sample to test the analyst's conjecture at a significance level of 0.10.

$\alpha = .10$

What variable was recorded? Is this variable categorical or quantitative?

regular snapchat use, Y/N $\qquad \hat{p} = \frac{438}{1066} = .4109$

Step 1

$H_0: p = .38 \qquad H_a: p > .38$ (use has risen)

Step 2

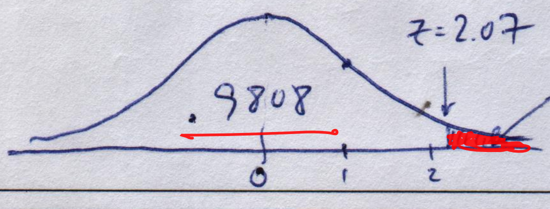
$\alpha = .10$: reject H_0 if probability $< .10$

Step 3

proportion (sample) $\hat{p}$ is $.4109$

$$T.S.\ z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.4109 - .38}{\sqrt{\frac{.38 \cdot .62}{1066}}} = 2.07\ldots$$

More than two std. dev. away | strange!



area we want:

$1 - .9808 = .0192 < \alpha$

probability of a z-score as extreme as 2.07 or more so!

Step 4

Decision: reject the null $H_0: p = .38$ in favor of the alternative $H_a: p > .38$

(because the probability $= .0192 < .10 = \alpha$)

We have enough evidence to believe that Snapchat use is up at the .10 level.

In order for the inference to be valid, what assumptions must be satisfied?

Normality & a random sample.

$np > 10$

$n(1-p) > 10$

$n \cdot p > 10$ ✓ ✓ Assume normal!

$p = .18$ $\quad n(1-p)$ is even bigger $\quad n = 700$

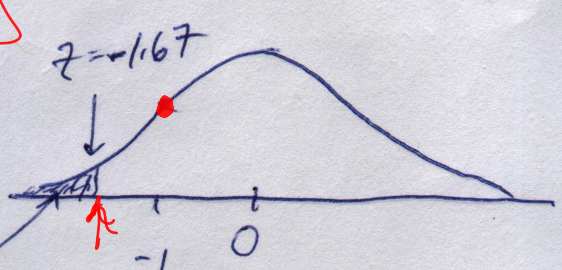
Example 2: In 2013, 18% of all Kentucky adults were uninsured. With the passing of the Affordable Healthcare Act, one believes that this percentage should decrease. To test this theory, a random sample of 700 KY adults is taken from driver's license records. It is found that 109 are uninsured. Use this sample to test the appropriate claim using a significance level of 0.05. $\alpha = .05$

Step 1: $H_0: p = .18 \qquad \boxed{H_a: p < .18}$

Step 2: $\alpha = .05$ $\quad$ Reject H_0 if prob. $< \alpha = .05$.

Step 3: $\hat{p} = 109/700 = \boxed{.1557}$

$\boxed{\text{T.S. } z = \dfrac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}} = \boxed{-1.67}$



Step 4:

Reject $H_0: p = .18$ in favor of $H_a: p < .18$

Probability of a z this extreme (or more so) $= .0475$ ✓

probability $= .0475 < .05 = \alpha$

What type of sample was taken? Would you feel comfortable generalizing the results of this survey to the entire population of US adults?

We have sufficient evidence to declare that the uninsured rate has gone down at the .05 level. ✓

$np > 10 \qquad 700 \cdot .18 > 10$ ✓

$n(1-p) > 10$

$p = .22$

Example 3: In 2011, Apple claimed that its online bookstore iBookstore had captured a 22% share of the US e-book market. To determine if there has been any fluctuation in the market, an analyst plans to take a random sample of 200 recent e-book purchases and determine the number of these purchases which were from iBookstore. $n = 200$

— Assuming there has been no change since 2011, determine if the assumptions satisfied for a hypothesis test to be conducted.

random sample ✓

normal? $\quad np = 200 \cdot .22 > 10$ ✓

$n(1-p) > 10$ ✓

part

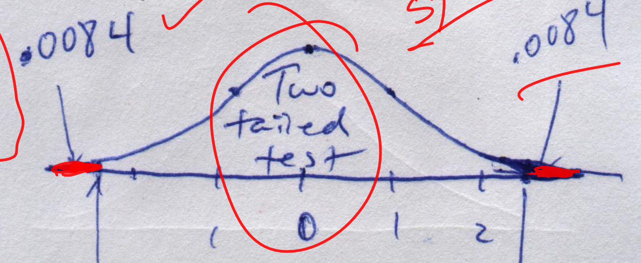
— When a random sample of 200 purchases was selected in 2012, it was found that 58 of these purchases were made using iBookstore. Test the appropriate hypotheses to determine if there has been any *change* in Apple's share of this market using a significance level of 0.01.

$\alpha = .01$ $\quad \hat{p} = \frac{58}{200} = .29$ $\quad n = 200$

Step 1: $H_0: p = .22$ $\quad H_a: p \neq .22$

Step 2: $\alpha = .01$ Reject H_0 if prob. $< \alpha = .01$.

Step 3: T.S. $z = \dfrac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \boxed{2.39}$



Total area = total probability $= .0168$ $\left(= \begin{matrix} .0084 + \\ .0084 \end{matrix}\right)$

$z = -2.39$ other tail as well. $z = \boxed{2.39}$

Step 4: $> \alpha = .01 = 1/100$

We fail to reject the null: We can't conclude that Apple's market share has changed.

$p = .065$

Example 4: In 2005, the CDC reported that 6.5% of parents of children (aged 0 – 17) have chosen to not vaccinate their child against one or more common childhood illnesses. In the past few years speculation has been raised that vaccinations could be linked to autism in children. Although this theory has been debunked by the CDC, a researcher has speculated that more parents are now choosing to not vaccinate their children. To test this conjecture, a random sample of 600 parents of children of vaccination age across the United States is selected and 54 of parents have children they have chosen not to vaccinate. Does the sample provide convincing evidence for the claim made by the researcher? Test the appropriate hypotheses using a significance level of 0.05.

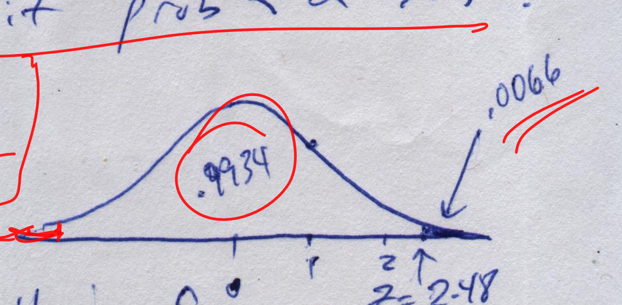
$n = 600$

$np = 600 \cdot .065 > 10$ ✓ $\quad \alpha = .05$ $\quad$ part $\quad \hat{p} = \frac{54}{600} = .09$

Step 1: $H_0: p = .065$ $\quad H_a: p > .065$

Step 2: $\alpha = .05$ So reject H_0 if prob $< \alpha = .05$.

Step 3: T.S. $z = \dfrac{(\hat{p} - p)}{\sqrt{\frac{p(1-p)}{n}}} = 2.48$



Step 4: prob $= .0066 < \alpha = .05$

So we reject the null in favor of the alternative: so we conclude that non-vaccination rates are up! at the .05 level.

Suppose the sample size were increased to 1500 individuals (but with the same sample proportion as used above). What would the new test statistic be? Would the probability associated with this new test statistic be larger, smaller, or unchanged from what was found in part b? **Explain.** $n=1500$ $\hat{p}=.09$

New test statistic: As n gets bigger, z gets bigger; as z gets bigger, the probability gets

Probability (circle one): Larger Smaller Unchanged

Explanation: The z value suggests how extreme the result: the more extreme, the less likely (lower probability).

Errors in Hypothesis Tests

Textbook pages: 543 – 550

In a hypothesis test, the decision is made from sample data, but a conclusion is made about a population parameter.

So, what's the problem??

The sample may not reflect the population (so $\hat{p}$ may not accurately reflect p)

Even with lots of evidence, the data can still lead to the wrong decision. When we perform a hypothesis test, we can make mistakes in one of two ways:

- Type I: reject a true null
- Type II: failing to reject a false null.

So, if errors are possible, why do we even bother with a hypothesis test?

There are two other cases: rejecting a false null or failing to reject a true one! Most of the time we get it right.

What effect does the significance level have on the errors?

α is the probability of a type I error. (The smaller α is, the harder it is to reject any null!)

Example: A drug manufacturer is developing an alternative to Prilosec for treating acid reflux. Suppose the published risk of headache when taking Prilosec is 3.8%. In a clinical trial of 1204 patients with acid reflux, 66 reported experiencing a headache after taking a daily dose of the drug.

- Identify the who, what, and why for this scenario.

$n = 1204$

$\hat{p} = \frac{66}{1204} = .055$

$p = .038$

- The drug manufacturer wants to know if the risk of headache with the new drug differs from that of Prilosec. What hypotheses should they test to see if there is evidence to support this conjecture?

$H_0: p = .038$ $\quad$ $H_a: p \neq .038$

- Using these hypotheses above, describe a Type I and Type II error. Also discuss the consequence of committing each of these errors from the company's perspective. From these consequences, what significance level do you feel should be set?

Type I: reject a true null: so $p = .038$ (no difference from Prilosec) but we conclude that there is a difference!

In fact

Type II: fail to reject a false null: $p \neq .038$, but we can't reject $p = .038$.

Nina lives in fear of a Type I error – so we'll choose a small α!

$\boxed{\alpha = .01}$

Reduces risk of Type I.

smaller → harder to reject a true null

(But inflates the risk of a Type II :)

- Now that a significance level has been chosen based on the consequences of the error, use the sample information given at the beginning of the problem to complete the hypothesis test.

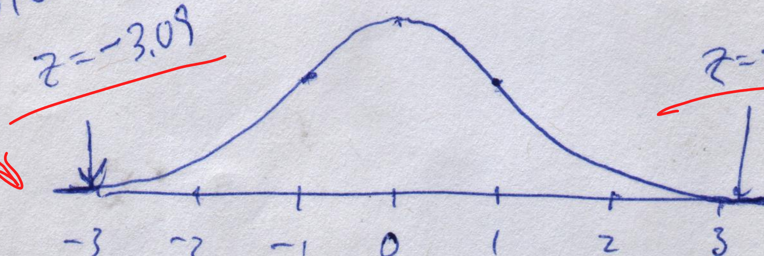
Step 1: $\boxed{H_0: p = .038}$ $\boxed{H_a: p \neq .038}$

Step 2: $\alpha = .01$ Reject H_0 if probability $< \alpha = .01$

Step 3: Test Statistic $z = \dfrac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \dfrac{.055 - .038}{\sqrt{\frac{.038(1-.038)}{1204}}} = 3.09$

area = .0010

$z = -3.09$



$z = 3.09$ by symmetry

area = .0010

.0020 < α

Total probability (of an z-score as extreme or more so than either $z = 3.09$ or $z = -3.09$) is $.0010 + .0010 = .0020 < .01 = \alpha$, so

Step 4: so we reject the null in favor of our alternative: we conclude that the evidence suggests that $p \neq .038$ at the $\alpha = .01$ level, & leaning toward our drug causing more headaches.