F1.4ZR1 **F1.4ZR1**
2003/04 **Special Topics 4 (Mathematical Biology)** 2003/04

Aims

To teach the applications of ordinary and partial differential equations to biology. Emphasis will be divided between the derivation of models for biological systems, new mathematical techniques for studying these models, and the biological interpretation of results.

Syllabus

Phase plane methods: Single species models. Review of phase plane methods for two coupled ODEs and an introduction to the use of nullclines. Survey of applications via a series of examples, including tumour formation, symbiotic systems, competing systems (illustrating competitive exclusion) and predator-prey systems (illustrating Hopf bifurcation). Introduction to modelling a biological system using ODEs. Non-dimensionalisation. (8 lectures)

Epidemiology: Develop simple models of infectious disease. Derive the threshold density for an epidemic. Derive the conditions for a disease to be endemic. Assess the relationship between disease persistence and R_0, the basic reproduction rate of the disease. Use the simple model results to highlight how vaccines can be used to eradicate disease. (3 lectures)

Wave fronts in reaction-diffusion equations: Introduction to reaction-diffusion models. Wave fronts in the Fisher equation; existence of minimum wave speed (sketch of proof). Wave fronts for scalar equation with cubic kinetics; existence of unique wave speed (sketch of proof), and derivation of wave form by inspection in simple cases. Applications in ecology and medicine. (6 lectures)

Spatial pattern formation: Reaction-diffusion equations as a model for developmental biology, in particular animal coat markings. Derivation of conditions for diffusion driven instability for two equations. Determination of possible patterns on finite and infinite domains in one and two space dimensions. Applications to patterning in ecology. (5 lectures)

Delay differential equations: Introduction to DDE models in ecology and physiology. Derivation of critical delay for stability in a single DDE. Construction of periodic solutions for piecewise constant negative feedback. (4 lectures)

Mathematical modelling: In addition to the above topics, the course will teach the process of deriving a mathematical model for a given biological situation. This is taught via a series of simple examples, spread throughout the course. (4 lectures)

Teaching and Assessment

Contact Hours: Term 1 (10 weeks): 3 lectures and 1 tutorial per week. Term 3: revision.

Final Mark = exam 100% + course work 0%.

Examined alone or with F1.4ZS2 and F1.4ZT3 (Special Topics 5 + Term 3 revision).

No Resit.

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By the end of the course, students should be able to:

- Give biological interpretations for the terms in simple ordinary and partial differential equation models for biological systems.
- Understand the mechanisms which determine species interactions.
- Determine steady states of biological models consisting of two coupled ODEs and their stability, and interpret the results biologically.
- Nondimensionalise simple ODE and PDE models.
- Understand simple infectious disease models and the concepts of epidemic, endemic and disease-free states.
- Derive R_0, the basic reproduction rate for a disease and relate it to vaccination strategies.
- Understand the law of mass action and its use in deriving reaction kinetic ODE models.
- Understand the way in which phase plane methods are used to study travelling wave solutions of scalar reaction-diffusion equations.
- Understanding the key differences between travelling wave solutions of scalar reaction-diffusion equations with quadratic and cubic kinetics.
- Derive travelling wave speeds for standard-type scalar reaction diffusion equations, and derive wave forms also in simple cases.
- Understand the process of diffusion driven instability.
- Derive conditions for diffusion instability for two coupled reaction-diffusion equations and use these to obtain parameter domains for DDI.
- Apply DDI conditions to derive forms of possible patterns in one-dimensional and two-dimensional (rectangular) domains.
- Understand the concept of a delay differential equation and the use of the delay as a bifurcation parameter, including calculation of the critical delay for stability.
- Construct explicit periodic solutions for a DDE with piecewise constant feedback.
- Write down mathematical models (ODEs or PDEs as appropriate) for simple biological situations.